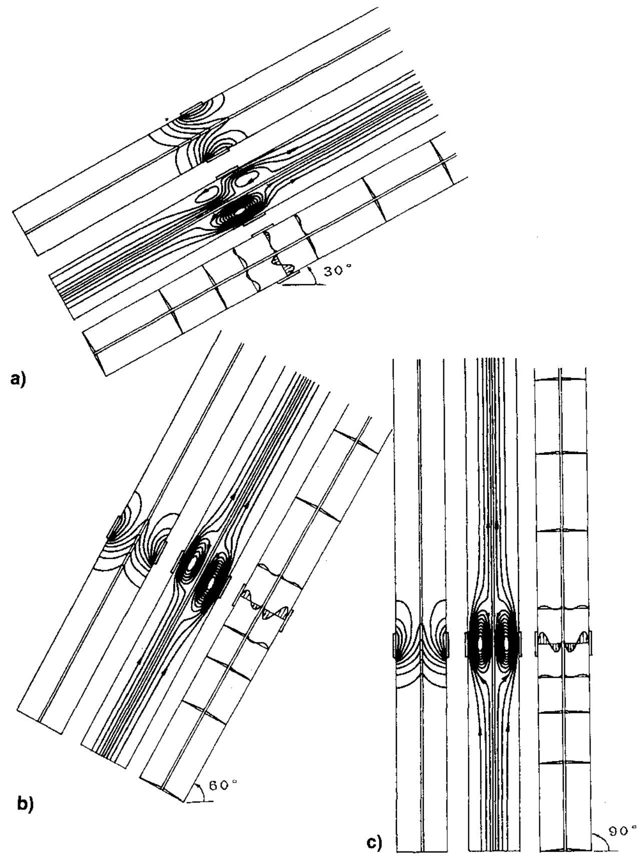

Fig. 3 Isotherms, streamlines, and velocity profiles for inclined duct at $Re = 1$, $Gr = 10^3$, $Pe_p = 7$, and $k^* = 8$: a) $\psi_{max} = 1.5587$, $\psi_{min} = -0.6683$, $\Delta\psi = 0.13$; b) $\psi_{max} = 1.7000$, $\psi_{min} = -1.1795$, $\Delta\psi = 0.13$; and c) $\psi_{max} = 1.5606$, $\psi_{min} = -1.5606$, $\Delta\psi = 0.13$.

Concluding Remarks

In this study, numerical simulations by means of a finite difference method have been performed to render the effects of the relevant physical parameters of the problem considered (Re, Gr, and ϕ) on the interaction between the shear-driven flow and the buoyant recirculating flow during the heating process of the moving plate. Results indicate that the shear-driven flowfields in the divided subchannels can be strongly affected by the buoyancy force due to an increase of Grashof number; a bicellular recirculation arises respectively within the subchannels in the region around the heaters. Moreover, an increase in Grashof number tends to promote the upstream diffusion effect due to the buoyant recirculation, particularly for the upper channel. Accordingly, the local Nusselt number on the top surface of the moving plate becomes less localized shifting in the upstream direction. Moreover, the flowfield and temperature distribution in the upper subchannel are found to be more sensitive to the inclination of the duct. For the range of inclination considered, the temperature as well as the heat transfer on the moving plate are rather unaffected by the variation of the duct orientation.

Acknowledgments

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References

[1]Sakiadis, B. C., "Boundary-Layer Behavior on Continuous Solid Surfaces: I. Boundary-Layer Equations for Two-Dimensional and Axisymmetric Flow," *AIChE Journal*, Vol. 7, 1961, pp. 26–28.

[2]Chen, T. S., and Strobel, F. A., "Buoyancy Effects in Boundary Layer Adjacent to a Continuous Moving Horizontal Flat Plate," *Journal of Heat Transfer*, Vol. 102, 1980, pp. 170–172.

[3]Karwe, M. V., and Jaluria, Y., "Fluid Flow and Mixed Convection Transport from a Moving Plate in Rolling and Extrusion Processes," *Journal of Heat Transfer*, Vol. 110, 1988, pp. 655–661.

[4]Karwe, M. V., and Jaluria, Y., "Numerical Simulation of Thermal Transport Associated with a Continuously Moving Flat Sheet in Materials Processing," *Journal of Heat Transfer*, Vol. 113, 1991, pp. 612–619.

[5]Ho, C. J., and Chang, J. Y., "Conjugate Natural Convection-Conduction Heat Transfer in Enclosures Divided by Horizontal Fins," *International Journal of Heat and Fluid Flow* (to be published).

Bias Error Reduction Using Ratios to Baseline Experiments—Heat Transfer Case Study

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Introduction

PRECISION uncertainties can be reduced by repeated trials and averaging. Bias or fixed uncertainties cannot be so reduced. In many engineering experimental programs, the ultimate result can practically be expressed as a ratio of two experimentally determined quantities. If experiments are conducted in a nominally identical manner using the same instruments, many of the bias uncertainties in the two experiments will be strongly correlated. When the results are presented as a ratio, these correlated bias uncertainties will tend to cancel. For bias-dominated experiments, the ratio can be considerably more certain than the individual measurements.

This scheme for bias error reduction can give considerable advantage when parametric effects are being investigated experimentally. As an example, consider a set of experiments where the effect of surface finish (riblets) on convective heat transfer is being studied. First, baseline experiments would be conducted with smooth surfaces. Then, a series of experiments would be conducted with different surface finishes, and a comparison of the heat transfer on each with the smooth baseline case would be made. The desired change in result is small and often on the same order as the uncertainties in the measurements. If the raw heat transfer data are compared directly, the small changes in results will be colored by the

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experimental uncertainty in the data. These small changes can be investigated with much greater confidence when the results are compared in terms of ratios with the baseline.

The bias uncertainties will cancel exactly only in cases where the two experiments are exact replications with perfect correlated biases. In fact, this bias reduction is not a general result, and pathological cases exist where the correlated biases can have an adverse effect on the uncertainty of the ratio. In more practical cases, such as the parametric study described above, the correlated bias uncertainties will not cancel exactly. However, even when the matching conditions are not pristine, considerable bias reduction can still be obtained. When results are presented as ratios with the hope of reducing the bias uncertainties, the effect of correlated biases should be carefully and methodically studied.

In this Note, a case study is presented for a series of heat transfer experiments. A more detailed presentation of this case is given by Chakroun et al.[1] The experiments are performed to study the effect of thermal boundary condition on heat transfer in the turbulent flat plate boundary layer. Three cases are considered: 1) a constant wall temperature case, 2) a constant wall heat flux case, and 3) a linear wall temperature distribution. The results are presented as the ratio of local Stanton number for each run to the local Stanton under nominally identical conditions, but with a constant wall temperature—symbolically $\eta = St/St_t$. The results are presented in this fashion so that they can be directly compared with theoretical solutions which are based on superposition of the unheated starting length problem.[2]

Experimental Apparatus and Measurements

The experiments were performed in the Turbulent Heat Transfer Test Facility which is a closed loop wind tunnel designed for boundary-layer heat transfer experiments. The description and qualification of this facility are given in the open literature[3] and are not repeated here.

For the baseline experiment, the temperature of each of the 24 smooth plates is measured by a thermistor and is controlled so that a constant wall temperature boundary condition is maintained. For the other experiments, the individual plate temperatures are controlled to maintain the desired wall temperature conditions. All of the experiments are conducted in the same facility with the same instruments. In this case, many of the bias uncertainties in the baseline and other experiments are perfectly correlated.

Data Reduction and Uncertainty Analysis

The Stanton number is determined by considering the energy balance on each individual test plate as shown in Fig. 1. Using the definition of Stanton number

$$St = \frac{W - (UA)(T_p - T_{\mathrm{rail}}) - (UAPP)(T_p - T_{pA}) - (UAPP)(T_p - T_{pB}) - \sigma\varepsilon A(T_p^4 - T_r^4)}{\rho C_p U_\infty A(T_p - T_0)} \tag{1}$$

The combined second, third, and fourth terms in the numerator represent the model for the conduction heat loss rate, and the last term in the numerator represents the model for the radiation heat loss rate.

The freestream air total and static temperatures are calculated using the measured recovery temperature and a recovery factor R for the probe $T_0 = T_r + (1 - R)U_\infty^2/2C_p$, $T_\infty = T_r - RU_\infty^2/2C_p$. The functional relationship for the moist air specific heat and for the moist air density are given by $C_p = C_p(T_\infty, T_{wb}, P_b, C_{p\mathrm{air}}, C_{pH_2O})$ and $\rho = \rho(T_\infty, T_{wb}, P_b)$, respectively. St_t is defined by the same formula with subscript t on each parameter. The ratio η is then given by

$$\eta = \frac{[W - (UA)(T_p - T_{\mathrm{rail}}) - (UAPP)(T_p - T_{pA}) - (UAPP)(T_p - T_{pB}) - \sigma\varepsilon A(T_p^4 - T_r^4)]}{[W_t - (UA)_t(T_{pt} - T_{\mathrm{rail}t}) - (UAPP)_t(T_{pt} - T_{pAt}) - (UAPP)_t(T_{pt} - T_{pBt}) - \sigma\varepsilon_t A_t(T_{pt}^4 - T_{rt}^4)]} \frac{\rho_t C_{pt} U_{\infty t} A_t(T_{pt} - T_{0t})}{\rho C_p U_\infty A(T_p - T_0)} \tag{2}$$

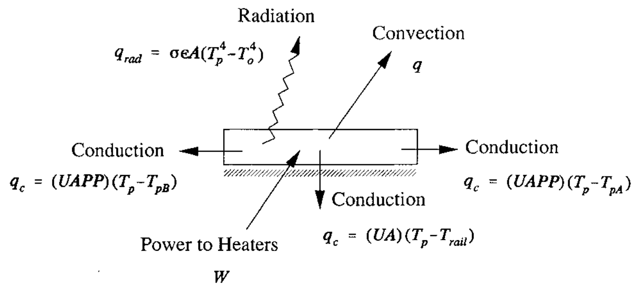

Fig. 1 Schematic showing the test plate energy balance.

In this experiment the precision uncertainties are negligible compared to the bias uncertainties. Using the uncertainty analysis procedures of Coleman and Steele,[4] the bias uncertainty in a result calculated from a set of measurements by $r = f(X_i, X_2, \ldots, X_N)$ is found from the uncertainty analysis expression

$$Br^2 = \sum_{i=1}^{N}\left[\theta_i^2 BX_i^2 + \sum_{k=1}^{N}\theta_i\theta_k B'X_i B'X_k(1 - \delta_{ik})\right] \tag{3}$$

The Kronecker delta is defined as $\delta_{ik} = 1$, when $i = k$ and $\delta_{ik} = 0$ when $i \neq k$. The term $B'X_i$ refers to the portion of the bias in X_i that arises from the same source as the bias in X_k. In the case of the thermistors, $B'X_i$ would include the elemental bias resulting from the bias in the calibration standard, but not that resulting from the thermistor installation. The sensitivity coefficients θ_i are defined as $\theta_i = \partial r/\partial X_i$.

For the Stanton number ratio [Eq. (2)], the biases for each of the common variables in the Stanton number, St and St_t, are 100% correlated. For example, the bias in T_p is 100% correlated with the bias in T_{pt}. In addition, all of the thermistor readings are partially correlated because they are all calibrated against the same standard. Therefore, the bias in T_p is partially correlated with the bias in T_{rt}, etc. All other correlation terms are considered to be zero.

A computer program is used to determine the uncertainty in the ratio η. The program gives also the uncertainty for each experimentally determined Stanton number separately. The data reduction equation is computed in a subroutine, and all the partial derivatives in the uncertainty equation [Eq. (3)] are approximated using finite differences.

Since precision limits in the measurements are assumed negligible, only the bias limits for the 32 variables are required. The variables, the bias limits, and the nominal values are shown in Table 1. The correlated bias errors in the temperatures resulting from the common calibration standard are

$$B'T_p = B'T_{pA} = B'T_{pB} = B'T_r = B'T_{\mathrm{rail}} = B'T_{pt}$$
$$= B'T_{pAt} = B'T_{pBt} = B'T_{rt} = B'T_{\mathrm{rail}t} = 0.04°\mathrm{C}$$

Results and Discussion

The results of the uncertainty analysis for the three heat transfer experiments are considered in detail in this Note. First, an actual replication case is considered, where a con-

Table 1 Bias limits and nominal values

Variable	St	St_t	Bias limit	Nominal values
Plate heater power	W	W_t	0.9%	20–150 W
Recovery temperature	T_r	T_{rt}	0.1°C	30°C
Plate temperature	T_p	T_{pt}	0.14°C	45°C
Rail temperature	T_{rail}	$T_{\text{rail}t}$	0.4°C	45°C
Wet-bulb temperature	T_{wb}	T_{wbt}	1.0°C	27°C
Plate area	A	A_t	0.03%	464.5 cm^2
Barometric pressure	P_b	P_{bt}	1.0 mmHg	760 mmHg
Specific heat of dry air	C_{pair}	$C_{\text{pair}t}$	0.5%	1.006 kJ/kg°C
Specific heat of water vapor	C_{pH_2O}	C_{pH_2Ot}	0.5%	1.86 kJ/kg°C
Freestream air velocity	U_∞	$U_{\infty t}$	0.4%	6–70 m/s
Recovery factor	R	R_t	0.09	0.86
Emissivity	ε	ε_t	0.05	0.11
Effective conductance (plate-rail)	(UA)	$(UA)_t$	45%	0.42 W/°C
Effective conductance (plate-plate)	$(UAPP)$	$(UAPP)_t$	50%	1.274 W/°C
Temperature of next plate	T_{pA}	T_{pAt}	0.14°C	45°C
Temperature of previous plate	T_{pB}	T_{pBt}	0.14°C	45°C

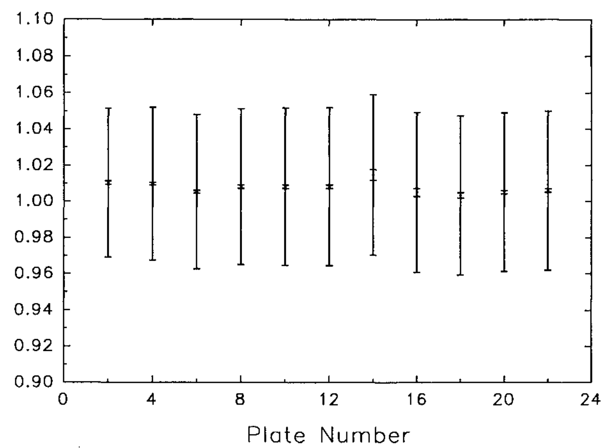

Fig. 2 Ratio of the Stanton number for the constant temperature case to that obtained from replicate run showing the overall uncertainty with and without the correlated terms.

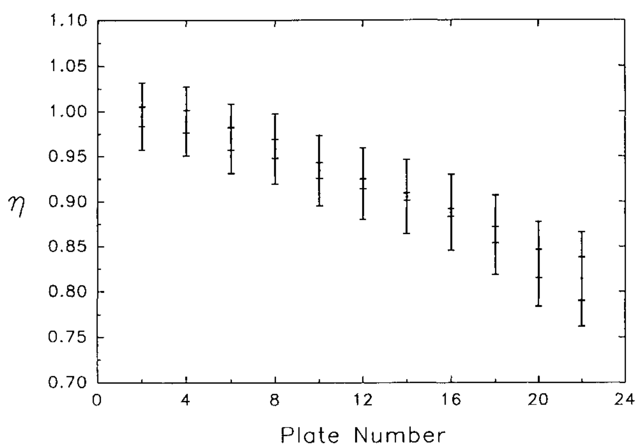

Fig. 4 Ratio of the Stanton number for the linear temperature distribution run to that of the constant temperature case showing the overall uncertainty with and without the correlated terms.

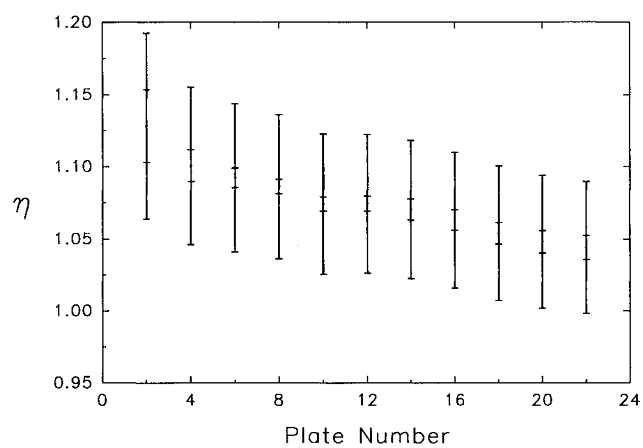

Fig. 3 Ratio of the Stanton number for the constant heat flux run to that of the constant temperature case showing the overall uncertainty with and without the correlated terms.

stant wall temperature boundary condition experiment is repeated twice. The second case considers the results for constant heat flux boundary condition, and the third case is a linearly decreasing wall temperature distribution. The three experiments are performed to study the effect of different wall thermal boundary conditions on heat transfer in the turbulent flat plate boundary layer. The constant wall temperature run is referred to here as the baseline case.

Figure 2 shows the distribution of the ratio of Stanton number for two replication tests where both runs are for constant temperature. Since this is an actual replication, the nominal values for each variable in St and St_t are close, but not exact. The values of η range from 1 to 1.02 along the 24 test plates. This small difference is due mostly to small drift in setting up the nominal conditions for each run. The large band of uncertainty is calculated without including the correlated bias terms between the variables of St and St_t. The small band shows the uncertainty in η after the bias terms in all the variables in St are correlated with those of St_t. A large reduction in the overall uncertainty is observed when the bias limits in the variables of St and St_t are correlated. The figure shows a significant reduction in the uncertainty when the results from the two experiments are presented as ratios. Also, the figure demonstrates that an assumption of all of the bias errors canceling is not strictly proper, but would be a reasonable approximation for this case.

Figure 3 shows a plot of the ratio of Stanton number for the constant heat flux case to that of the constant temperature run along all 24 test plates. The figure presents the uncertainty limits calculated with and without correlating the bias limits in the variables of St and St_t. The large uncertainties are obtained when the bias limits in the variables of St are not correlated with those of St_t. The calculation for the small uncertainty includes all correlated terms in St and St_t. A large reduction in the uncertainty in η is observed when the bias errors in all the variables of St and St_t are correlated. However, in this case it would be a poor approximation to assume that all bias errors cancel.

Figure 4 presents a plot for the ratio of Stanton number for the linear temperature distribution run to that of the constant wall temperature baseline case. The values of η range from 1 to 0.83. Again, a large reduction in the overall uncertainty in η is seen when the bias limits in the variables of St and St_t are correlated. In this case a considerable amount of uncertainty remains in η, and it would be improper to assume that all biases cancel.

When the results of an experiment are presented as a ratio with the baseline results, a large reduction in the overall uncertainty can be achieved when all the bias limits in the variables of the experimental result are fully correlated with those of the baseline case. This case study demonstrates the methodical treatment of correlated bias errors for ratios and demonstrates the power of this technique to reduce bias uncertainty in such experiments.

References

[1]Chakroun, W. M., Taylor, R. P., Steele, W. G., and Coleman, H. W., "Bias Error Reduction in Experimental Results by Presentation as a Ratio to a Baseline Experiment—A Heat Transfer Case Study," AIAA Paper 93-0922, Jan. 1993.

[2]Kays, W. M., and Crawford, M. E., *Convective Heat and Mass Transfer*, McGraw-Hill, New York, 1980.

[3]Coleman, H. W., Hosni, M. H., Taylor, R. P., and Brown, G. B., "Using Uncertainty Analysis in Debugging and Qualification of a Turbulent Heat Transfer Test Facility," *Experimental Thermal and Fluid Science*, Vol. 4, No. 6, 1991, pp. 673–683.

[4]Coleman, H. W., and Steele, W. G., *Experimentation and Uncertainty Analysis for Engineers*, Wiley, New York, 1989.